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Numerical modeling of the KdV random wave field

Efim Pelinovsky [a,b,*], Anna Sergeeva (Kokorina) [a,b]

[a] Laboratory of Hydrophysics and Nonlinear Acoustics, Institute of Applied Physics, Nizhny Novgorod 603950, Russia
[b] Applied Mathematics Department, Nizhny Novgorod State Technical University, Nizhny Novgorod 603950, Russia

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Abstract

The evolution of the initially random wave field with a Gaussian spectrum shape is studied numerically within the Korteweg–de Vries (KdV) equation. The properties of the KdV random wave field are analyzed: transition to a steady state, equilibrium spectra, statistical moments of a random wave field, and the distribution functions of the wave amplitudes. Numerical simulations are performed for different Ursell parameters and spectrum width. It is shown that the wave field relaxes to the stationary state (in statistical sense) with the almost uniform energy distribution in low frequency range (Rayleigh–Jeans spectrum). The wave field statistics differs from the Gaussian one. The growing of the positive skewness and non-monotonic behavior of the kurtosis with increase of the Ursell parameter are obtained. The probability of a large amplitude wave formation differs from the Rayleigh distribution.

Keywords: KdV equation; Random waves; Distribution function; Spectrum

1. Introduction

The Korteweg–de Vries equation is now a fundamental model of the weakly nonlinear waves in the weakly dispersive media. Initially it was derived for surface waves in shallow water, and then – for different waves in incompressible and stratified fluid, magneto hydrodynamics, plasma, electrical transmission lines, etc. [1–5]. Due to a full integrability, this equation demonstrates an important role of the solitary waves (solitons) in the nonlinear wave dynamics. For initial disturbances vanishing at infinity, the solitons present the final stage of the wave field evolution, and these results are well known. If the initial disturbance presents the sine periodic wave, its evolution leads to the soliton formation and its disappearance (recurrence phenomenon), as it has been shown in the pioneer work by Zabusky and Kruskal [6]. Then this process has been investigated for an arbitrary ratio of nonlinearity to dispersion (Ursell parameter) and large times, see, for instance, the recent papers [7–9] and papers referenced there. Actually, an initial sine state is not fully reconstructed in large time, and soliton ensembles play an important role in the long-time behaviour of a nonlinear wave field, namely for large values of the Ursell parameter. The dynamics of the soliton ensembles even for this simple sine initial condition is very complicated, and, perhaps, it can be interpreted as a soliton turbulence,

* Corresponding author. Tel.: +7 8312 164839; fax: +7 8312 365976.
E-mail address: pelinovsky@hydro.appl.sci-nnov.ru (E. Pelinovsky).

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which can be considered as a combination of "rarified solitonic gas" and residue oscillating quasilinear waves [10]. Zakharov [11] has used the inverse scattering method and has shown that only pair collisions occurred in the soliton ensemble, and the interaction with non-soliton field could not change the amplitude or phase of the soliton. As a result, the total soliton velocity distribution function does not depend on time.

Generally speaking, the same conclusion can be done for any periodic initial disturbances. The periodic solution of the Korteweg–de Vries equation is expressed through the theta-functions (see, for instance, [12–17]) and is represented by a linear superposition of nonlinear oscillatory modes (multi quasi-cnoidal waves) in the associated spectral problem ("nonlinear" Fourier series). The number of such modes is not too big as in the Fourier method, and this leads to an effective analysis of the wave field (including random sea state) and selection of soliton components. The parameters of the nonlinear oscillatory modes do not change in the process of the wave evolution, and in this sense, a distribution function of the corresponding parameters does not depend on time.

Meanwhile, the nonlinear energy exchange between different spectral components even for initial narrow-band wave field is significant: a wave packet splits into several groups with different carrier wave numbers, and a wave profile becomes asymmetrical [18–21]. A wave realization, presented by the multi-cnoidal waves and solitons, varies in space and time more significantly, and its behaviour is irregular (quasi-chaotic). Moreover, if an initial spectrum has two peaks, such state is unstable [11,22], and therefore the wave dynamics should be complicated. As a result, the statistical moments and the distribution functions of the wave field will change in time; its spatial spectrum will vary also. Large-amplitude pulses (freak waves) will appear and disappear in the process of the wave evolution [23–25]. On the assumption of the random initial conditions, the properties of such wave field can be studied with relation to the theory of random functions. In fact, we know the only mathematical paper [26] when the soliton generation from irregular data is studied, but it does not refer to the random wave field.

The main goal of given paper is to study numerically the evolution of an initially random wave field within the Korteweg–de Vries equation. The method of the direct simulation of the simplified models without using of any turbulent hypothesis is now popular in the fluid mechanics, and we would like to point here the study of random waves within the nonlinear Schrödinger and Dysthe equations [27–29], Benney–Luke system [30], Zakharov equation [31–34] and model nonlinear-dispersive equation [35–37]. It is usually used to compare direct results with predictions of the weak turbulence theory [38,39]. As far as we know, the weak turbulence theory for the Korteweg–de Vries equation has not yet developed, and direct numerical simulation here can allow to understand the properties of the KdV random wave field. The following properties will be studied in given paper: spectrum evolution and stabilization, the third and the fourth statistical moments of the random wave field, and the distribution function for the crest amplitudes.

2. Basic model

The Korteweg–de Vries equation is considered here as a mathematical model for unidirectional surface gravity weakly nonlinear and weakly dispersive waves

$$\frac{\partial\eta}{\partial t}+c\left(1+\frac{3}{2h}\eta\right)\frac{\partial\eta}{\partial y}+\frac{ch^2}{6}\frac{\partial^3\eta}{\partial y^3}=0 \tag{1}$$

where $\eta(y,\tau)$ is a surface elevation, h is a water depth assuming being constant, g is a gravitational acceleration, $c=(gh)^{1/2}$ is a maximum velocity of a long wave propagation, y is a horizontal coordinate and τ is a time. Initial conditions for (1) can be written in a schematic form

$$\eta(y,0)=A_sF(Kk_{\max}y)\sin(k_{\max}y) \tag{2}$$

where A_s is a characteristic wave amplitude (for random wave field it is a significant wave amplitude A_s equal to 2σ, σ^2 is a variance), $k_{\max}$ is a carrier wave number (for random wave field it is the spectral peak wave number), and $F(Kk_{\max}y)$ is a wave envelope with the characteristic dimensionless spectrum of width K. Introducing the dimensionless variables

$$\xi=\frac{\eta}{A_s},\quad x=k_{\max}(y-ct),\quad t=\frac{3ck_{\max}A_s\tau}{2h} \tag{3}$$

Eq. (1) reduces to the dimensionless Korteweg–de Vries equation

$$\frac{\partial\xi}{\partial t}+\xi\frac{\partial\xi}{\partial x}+\frac{1}{9Ur}\frac{\partial^3\xi}{\partial x^3}=0 \tag{4}$$

where

$$Ur = \frac{A_s}{h^3 k_{\max}^2} \tag{5}$$

is the Ursell parameter. This parameter characterizes the relative role of nonlinearity and dispersion: small values of Ur correspond to the linear problem described by the Airy equation derived from (4); while large values of Ur correspond to the nonlinear hyperbolic long wave equation. The second parameter arises in the initial condition (2), it is the relative spectrum width, K. So, the dynamics of the wave field in general case is determined by two dimensionless parameters: Ur and K.

The numerical integration of the Korteweg–de Vries equation (4) with the periodic boundary conditions: $\xi(0,t) = \xi(L,t)$ is based on the pseudospectral method [40]. A zero-mean random wave field is modeled as the Fourier series and contains 256 harmonics

$$\xi(x,0) = \sum_{i=1}^{256} \sqrt{2S(k_i)\Delta k}\,\cos(k_i x + \varphi_i), \tag{6}$$

where $S(k)$ is an initial power spectrum, $k_i = i\Delta k$ and Δk is a sampling wave number, varied from 0.03 to 0.023, and a phase φ_i is a random variable, uniformly distributed in the interval $(0, 2\pi)$. The length of initial realization is $L = 2\pi/\Delta k$. An initial power spectrum is assumed to have a Gaussian shape of the amplitude, Q, and the width, K.

$$S(k) = Q\exp\left(-\frac{(k-1)^2}{2K^2}\right), \tag{7}$$

and parameter Q is chosen so, that $\int 2S(k)\,\mathrm{d}k = (s_0)^2 = 1/4$ (s_0 dimensionless value of a variance). In the case $K \ll 1$ it is calculated explicitly, $Q = 1/8K(2\pi)^{1/2}$. The dimension of spectral domain (256 harmonics) of ten characteristic spectrum widths is chosen to provide a spectrum decay in the region of large k. The initial spectra with a cut off spectrum tail are presented in Fig. 1.

In our numerical experiments the Ursell parameter varies from 0.07 to 0.95, and spectrum width varies from 0.27 to 0.18. The statistical characteristics are computed for each time step and are averaged over the 500 ensembles, what corresponds to a total wave record containing about 15 000 individual waves to provide a sufficient statistics. For example, Dysthe et al. [29] used ensemble of 50 records (2500 waves) to describe a spectrum evolution; Janssen [41] considered the size of the ensemble of 500 members; recently, Onorato et al. [42] studied statistical characteristics in comparison with experimental data and used an ensemble of 1000 time series. Our main aim is to determine the time evolution of the spectrum and distribution functions (water elevation and crest amplitude). Also, the skewness, m_3, and kurtosis, m_4, of a random wave field are calculated (note, that the first two moments: mean level and variance, are

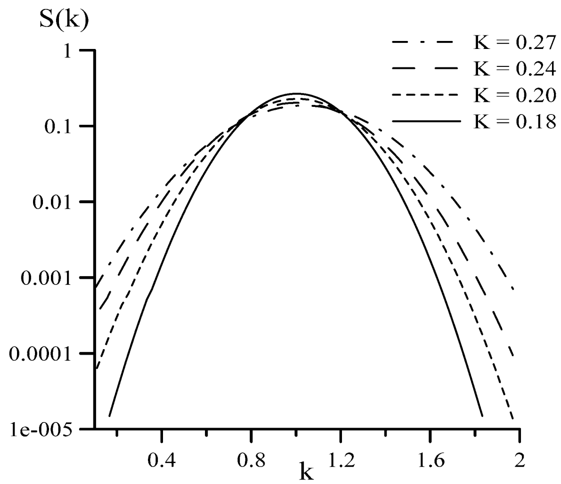

Fig. 1. Initial spectra for different width K.

the integrals of the Korteweg–de Vries equation, and they are constant in a process of wave evolution). For skewness and kurtosis the standard definitions are used:

$$m_3 = \frac{M_3}{s_0^3}, \qquad m_4 = \frac{M_4}{s_0^4} - 3 \tag{8}$$

where $s_0^2 = 0.25$, M_3 and M_4 are the second, the third and the fourth statistical moments of the dimensionless zero-mean wave field correspondingly and are defined through the surface displacement as

$$M_n = \frac{1}{N}\sum_{k=1}^{N} \xi_k^n. \tag{9}$$

As it is known, the skewness is a statistical measure of the vertical asymmetry of the wave field, and its sign defines the ratio of crests to troughs. If it is positive, the crests are bigger then the troughs. The kurtosis represents a degree of the peakedness in the distribution and defines the contribution of big waves in a total distribution. If it is positive, then the contribution of the big waves is rather significant.

After normalization the KdV equation (4) has the time scale as the nonlinear time and this is simple seen from (4) deleting the dispersion term. The computing is done for relative large nonlinear time equal to 100 and includes about 1000 wave periods, depending on the initial conditions, and this time is enough for expressing of nonlinear and dispersive effects and to reach equilibrium conditions. For instance, in simulations by Dysthe et al. [29] computed time is 100–200 wave periods, Onorato et al. [42] – 300 wave periods.

3. Results

The first experiment is set for different values of the Ursell parameter, but with constant spectra width $K = 0.27$.

Wave field. The evolution of the wave record ($Ur = 0.73$, $K = 0.27$) is displayed in Fig. 2 for different time moments. As it is immediately seen that the wave profile becomes asymmetric, what corresponds to sharp crests and gentle troughs. It means that skewness exceeds its initial value. The more details about the moment behavior will be given below. It is interesting to analyze the trajectory patterns (Fig. 3), presented in time–space plane, for the purpose of solitons existence for different initial conditions. The number of visible solitons is about 5 for $Ur = 0.95$ and there is only one visible soliton for $Ur = 0.2$. It means that the solitons do not contribute significantly in the total random field. Under condition of strong nonlinearity the propagation gives rise of a peak amplitude (Fig. 4(a), where gray line corresponds to the maximum amplitudes in the case of big Ur number, and black one – to the case of a weak nonlinear wave field propagation). The comparison with linear propagation (nonlinear term is cancelled), which is presented in Fig. 4(b) as the distribution functions of the largest amplitudes, well demonstrates the role of nonlinear effects in formation of large wave amplitudes.

Moments. The evolution of statistical moments shows the stationary state existence and transition to it. Transition period is about 10–20 nonlinear times, and during this period both moments of the wave field tend to the almost constant values (Fig. 5). Fig. 6 displays the values of m_3 and m_4, corresponding to this stationary mode.

For all the conditions, the skewness is positive, and it means that the positive waves (crests) have larger amplitudes then the negative waves (troughs). The asymptotic value of skewness increases with increase of the Ursell parameter; and, therefore, elevation (positive) waves are more visible in the nonlinear wave field then the depression (negative) waves. This conclusion corresponds to the known expressions for the Stokes waves [1].

The kurtosis tends to the negative asymptotic value for $Ur < 0.6$; therefore, the probability of the large amplitude wave (freak wave) occurrence should be less than it is predicted for the Gaussian processes. In the case of strong nonlinearity, the kurtosis asymptotic value exceeds zero level, what indicates a high probability of the large wave appearance. For deep water, as it was already shown by Onorato et al. [27] and Tanaka [31], the calculated values of kurtosis oscillate around zero with some exceeding for strong nonlinear random wave process. Janssen in [41] demonstrated a positive fourth moment, calculated in the weak turbulence theory for deep-water waves, which grows while the wave amplitude increases. So, the behaviour of the fourth moment is qualitatively the same in deep and shallow water.

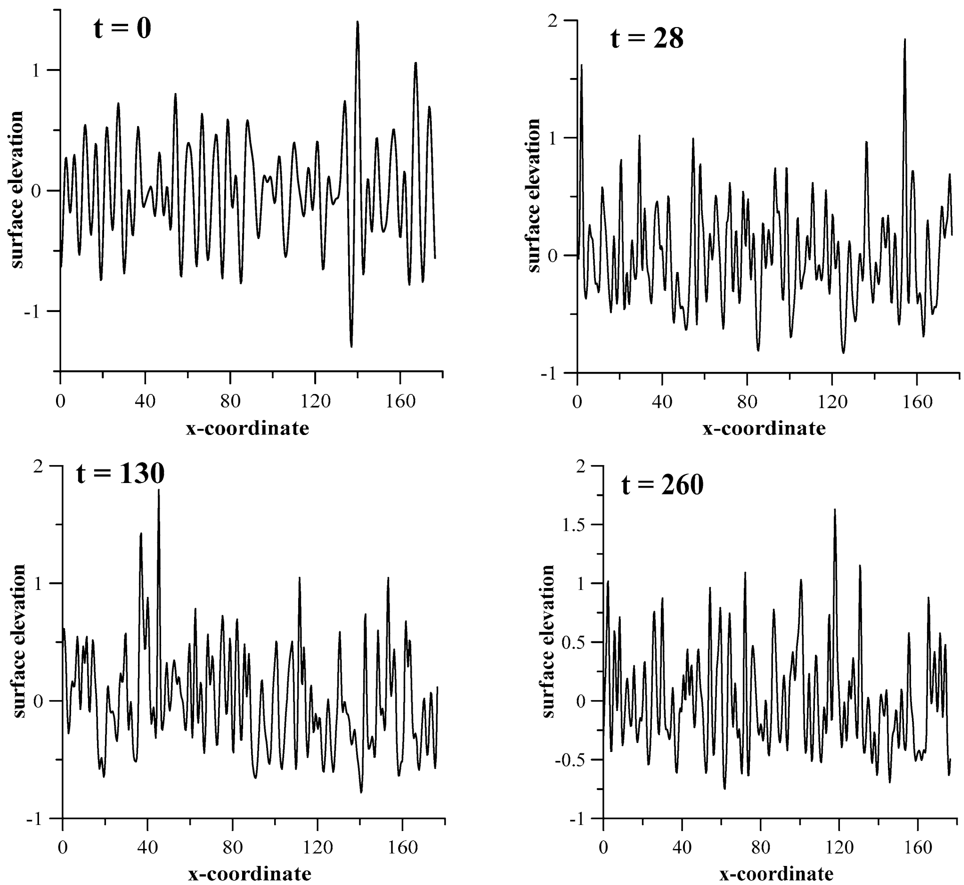

Fig. 2. Wave profiles at different time moments. $Ur = 0.73$, $K = 0.27$.

Spectrum width. Similar calculations are done for various values of the spectrum width K, varying from 0.18 to 0.24. The obtained results are compared with the previous one and computed curves of the both of calculations are demonstrated in Fig. 7, where full symbols correspond to the experiment with various $K \approx 0.18$–0.24, and empty symbols correspond to the constant width $K = 0.27$. The evolution of statistical characteristics in this case has the same tendency, as in the previous experiment.

Spectra. Firstly the Gaussian spectrum of constant width $K = 0.27$ is taken as a initial condition and its evolution is considered for different Ur, varying from 0.07 (almost linear evolution) to 0.95, what corresponds to a strong nonlinear wave field. As it is expected, due to nonlinearity the spectrum transforms, widens and tends to a stationary state (Fig. 8). This state, depending on the Ursell parameter, corresponds to the asymmetric shape: the main energy transfers into the low frequencies (spectrum downshift effect). For large Ursell values (Fig. 8(b)–(d)) the spectral density is distributed almost uniformly in small k. The flatness of the spectrum is bigger for $Ur = 0.95$, when the wave field is more energetic and nonlinear effects are more significant. Tendency to the flatness of spectrum is known for the statistical equilibrium with no sources and sinks. The Rayleigh–Jeans spectrum, describing this equilibrium state, is a solution of the weak turbulence theory [38] and here we illustrate how this equilibrium is reached up for large values of the Ursell parameter. We would like to mention that the KdV equation has an infinite number of conservation quantities and correspondingly an infinite amount of stationary spectra; the Rayleigh–Jeans distribution is just one of these spectra. By the way, computed spectrum can be interpreted as the Rayleigh–Jeans distribution. It is important to mention that the spectrum is downshifted in low-frequency range even for initial spectrum of the symmetric Gaussian form. For comparison, the downshift of initial symmetric spectrum for deep-water waves is possible only in extended version of the nonlinear Schrödinger equation like the Dysthe equation, which includes

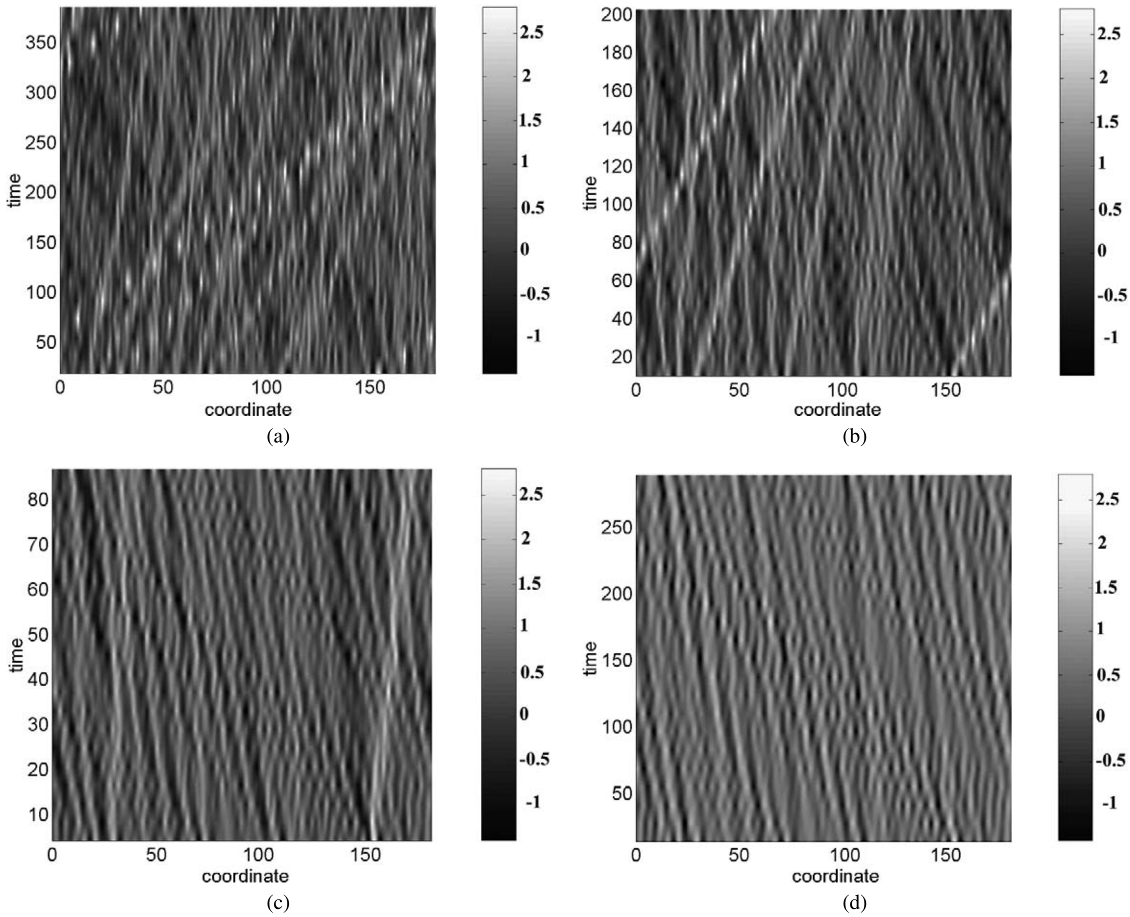

Fig. 3. Time–space plane of wave propagation for various *Ur*: (a) 0.95; (b) 0.5; (c) 0.2; (d) linear evolution.

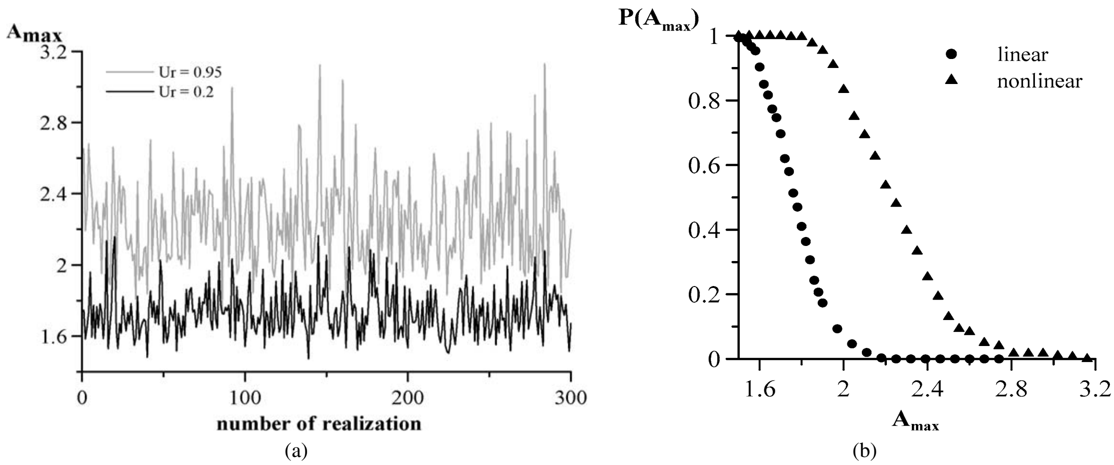

Fig. 4. (a) – maximum of wave amplitudes over time; gray line corresponds to the case of big *Ur* number, and black one – to the case of a weak nonlinear wave field propagation; (b) – distribution of maximum crest amplitudes over 300 realizations for nonlinear and linear wave filed propagation.

an asymmetry of the wave field [29]. The shallow water model based on the Korteweg–de Vries equation is initially asymmetric due to a quadratic nonlinearity, and the asymmetry of the wave group is immediately obtained in the process of the wave evolution [18–21].

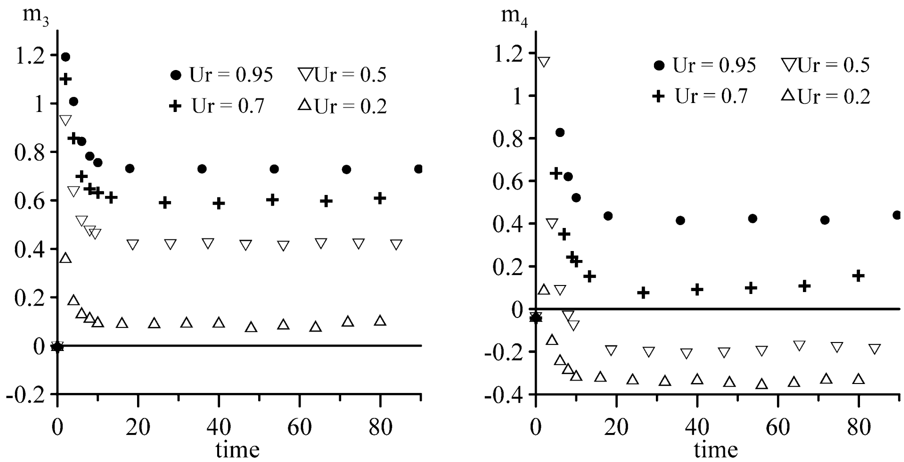

Fig. 5. Temporal evolution of skewness for different Ursell parameters.

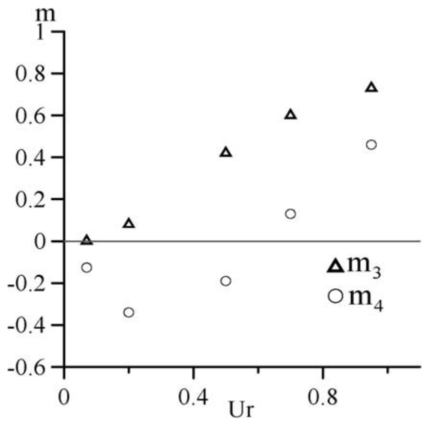

Fig. 6. Asymptotic values of the moments as a function of the Ursell parameter *Ur*.

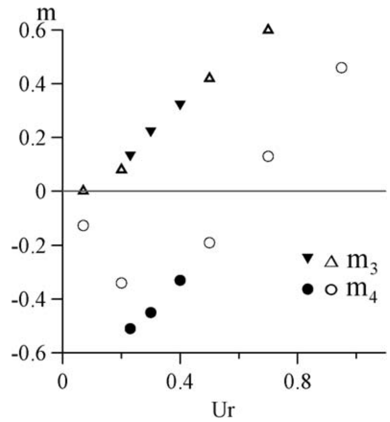

Fig. 7. Asymptotic values of the moments, depending on the Ursell parameter. Full symbols correspond to the experiment with variable spectrum width K from 0.18 to 0.24.

In the case of narrower initial spectrum ($K \sim 0.24$–0.18) wavelength becomes smaller and dispersive effects dominate. At that, spectral dilatation slows down, and weak harmonic generation both in low- and high-frequency regions is noticed with K decrease (Fig. 9).

As it has been already noticed, spectrum becomes asymmetric with weak shifting in short wave range. For larger k ($0.1 < k < 0.2$) the slope of the spectrum decreases with increase of the Ur parameter (from 3.7 for $Ur = 0.5$ till $\alpha = 2$ for $Ur = 0.95$).

Distribution function. Distribution of the wave crest amplitudes, calculated as a maximum between two zeros, is demonstrated in Fig. 10. The results are compared with the theoretical Rayleigh distribution (integral distribution function) of the amplitudes of the narrow-band Gaussian process

$$P(A) = \exp(-2A^2). \tag{10}$$

For the case of $Ur < 0.3$, the probability of small amplitudes ($A < 1.2$) exceeds the Rayleigh distribution, meanwhile in the range of high amplitudes ($A > 1.5$) the distribution lays below theoretical curve. And for more energetic wave field ($Ur > 0.3$) the asymptotic distribution exceeds the Rayleigh distribution, and the probability of the highest crests appearance increases. In qualitative sense, the shape of the amplitude distribution function is not in contradiction with the behavior of the skewness and kurtosis (Fig. 5); the first one shows the positive waves have larger

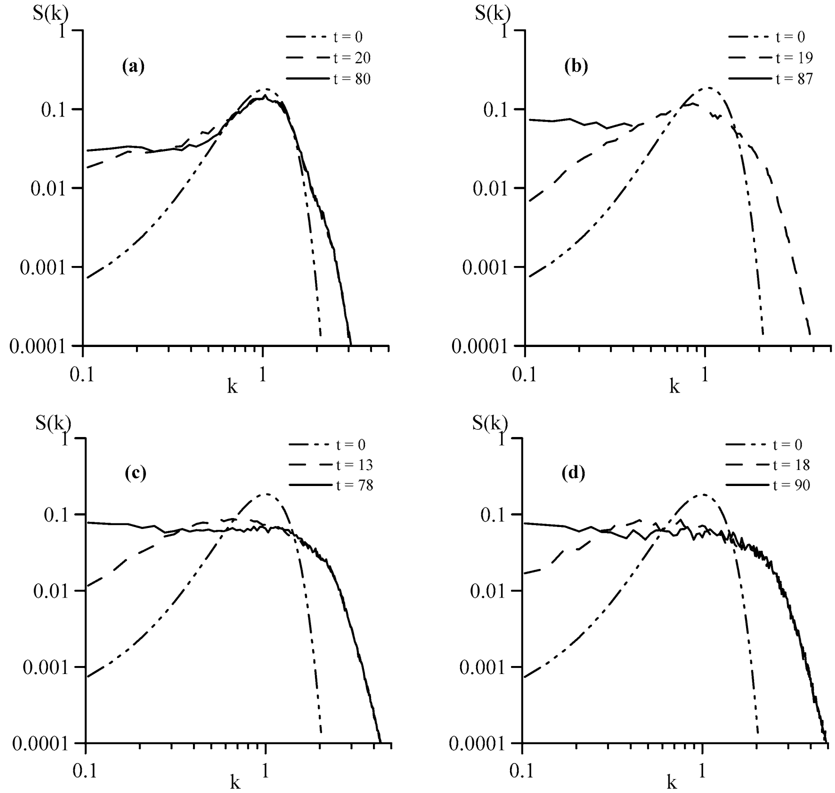

Fig. 8. Temporal evolution of spectra $S(k,t)$ for various Ur: (a) 0.2, (b) 0.5, (c) 0.7, (d) 0.95.

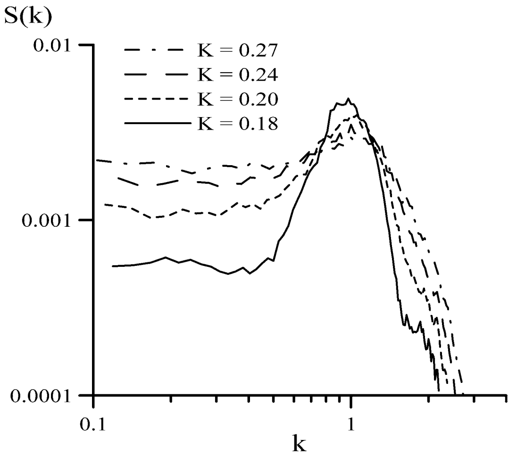

Fig. 9. Stationary spectra for different values of the initial spectrum width K.

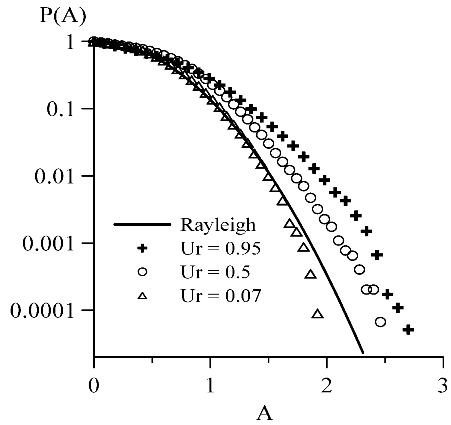

Fig. 10. Asymptotic crest amplitude distribution for different Ur numbers. Solid line corresponds to the Rayleigh distribution of the narrow-band Gaussian process.

amplitudes than negative waves, meanwhile the second one indicates a significant contribution of the small waves in a total distribution. Finally, these results allow to estimate the probability of the freak wave formation (its amplitude exceeds double significant amplitude by definition). Freak waves should appear more frequently if the wave field is more nonlinear (high values of the Ursell parameter).

4. Conclusion

The dynamics of a random wave field with a Gaussian statistics is numerically studied within the Korteweg–de Vries (KdV) equation. The obtained results confirm that irregular wave field, taken as a result of wave evolution in the frame of the KdV model, does not satisfy the Gaussian statistics, and its statistical characteristics depend on the Ursell parameter, which defines a ratio of nonlinear effects to dispersion. In such a way it is demonstrated that the wave field becomes asymmetric: it is sharpened at the crests and flattened at the troughs, what leads to the positive third moment. The coefficient of skewness grows monotonously with increase of the Ur number. The behavior of the 4th statistical moment (kurtosis) is non-monotone. Its values are negative while $Ur < 0.8$, what indicates the significant distribution of small waves in a total distribution. If initial disturbance is more nonlinear, then kurtosis exceeds zero level. At that, it increases with Ur growth. In the case of small Ur numbers, closed to zero, the probability distribution function slightly deviates from the theoretical Rayleigh distribution. For $Ur > 0.3$ the computed curve lies above theoretical distribution, what means the higher probability of large wave formation, particularly the freak wave appearance.

An important result is the demonstration of the steady state existence for statistical characteristics: statistical moments (skewness and kurtosis), distribution functions and also spectral density. The computations demonstrate, that both of statistical moments and distribution functions evolve till some bound level is reached up. The analysis of a random wave spectrum evolution shows the same effect. The initially symmetric power spectrum with a Gaussian shape broadens in time with energy transference down the spectrum. During the time, approximately equal to 20 of a characteristic nonlinear time, the spectrum relaxes to some stationary state with energy concentration in low frequency range, as it has been already noticed. The parameters of the equation, in particular, the Ur parameter influences the width of the steady spectrum: in the case of strong nonlinearity the established stable spectrum is wider, and energy is distributed almost uniformly in the range of long waves.

Acknowledgements

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